Omissions and Measurement

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Abstract

Ayn Rand famously said that measurement omission is an essential part of concept formation. This essay will challenge that. It will be argued that something else is omitted much, even most, of the time. Since a clear understanding of the nature of measurement is a prerequisite for this argument, such nature will be explored. This will include its meaning in science, comparing it to ordinal ranking, and young children's ability to measure. It will end with possible substitutes for "measurements omitted" and other mathematical perspectives.

Introduction

Ayn Rand wrote, "A concept is a mental integration of two or more units possessing the same distinguishing characteristic(s), with their particular measurements omitted" (Rand 1990, 13). She did not include quantifiers. She did not literally say that *every* omission is of measurement(s) and it is *only* measurements that are omitted. However, this recognizes only the main part of ITOE. Per the expanded second edition, she later said, "to establish the similarity by showing the characteristic is the same and *only* [emphasis mine] the measurements vary" (ibid., 221).

I agree with her more general claim that concepts are grounded in similarities *and differences*, as elaborated in A Theory of Abstraction (Kelley, 1984). Kelley does an admirable job of elaborating Rand's framework. However, I contend that the framework is partly flawed, since it claims that all differences between similar things are ones of measurement.

Her claim was preceded by the example *length*. "If a child considers a match, a pencil, and a stick, he observes that length is the attribute they have in common, but their specific lengths differ. The *difference is one of measurement*. In order to form the concept 'length', the child's mind retains the attribute and omits its particular measurements" (ibid., 11).

She gave only one other example -- the concept *table* -- before stating her claim. After giving a definition for it, she says the definition retains and specifies the characteristic shape. The particular shape of the horizontal surface – round, square, or other – and its dimensions are omitted. Also omitted are the number and shape of the legs, the material of which the table is made and its utilitarian purpose. Attempting to explain how measurements are omitted with regard to the material, she says the measurements that differentiate one material from another. Similarly with regard to purpose, she says the utilitarian requirements set limits on the dimensions of the table and rule out unsuitable materials, such as non-solids (ibid., 12).

Were her justifications for material and purpose adequate? Is there some unit of measurement that differentiates between wood, plastic, steel, glass, and so forth, or combinations of different materials? Is there one that differentiates between the purposes of a dining table, an end table, a dressing table, a work table, and so on? In my view material and purpose are only further differentiated by classifying, not measuring. Did she consider examples of a wide enough variety to justify her general claim of measurement omission? I think the answer to each of these questions is 'no'.

Many pages later Rand discusses concepts of consciousness. There she undercuts her previous claim of omitting only measurements.

> For instance, the concept "thought" is formed by retaining the distinguishing characteristics of the psychological action (a purposely directed process of cognition) and by omitting the particular contents as well as the degree of the intellectual effort's intensity. The concept "emotion" is formed by retaining the distinguishing characteristics of the psychological action (an automatic response proceeding from an evaluation of an existent) and by omitting the particular contents (the existents) as well as the degree of emotional intensity (ibid., 32).
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> These concepts are formed by retaining their distinguishing characteristics and omitting their content. For instance, the concept "knowledge" is formed by retaining its distinguishing characteristics (a mental grasp of a fact(s) of reality, reached either by perceptual observation or by a process of reason based on perceptual observation) and omitting the particular fact(s) involved (ibid., 35).

Why is it omitting particular "contents" and "facts"? What happened to omitting only *measurements*? Moreover, there are more than measurements omitted for many concepts beside concepts of consciousness, especially for higher level concepts. More will be said on this later. It is time to consider the nature of measurement.

Measurement

Aristotle briefly talked about measurement as follows:

> We call a quantity that which is divisible into constituent parts of which each is by nature a one and a 'this'. A quantity is a *multitude* if it is numerable, a *magnitude* if it is measurable (Metaphysics, V, 13, 1020a 7-10).

The terms 'measurement', 'measure', and 'magnitude' are closely related. Sometimes they are used interchangeably. The glossary from Michell (1999) has the following.

> **magnitude** a specific level of quantitative attribute (or quantity). For example, each specific length that any object might have is a magnitude of the attribute, length.
> **measure** an estimate of the ratio of a magnitude of a quantity to a unit of the same quantity.
> **measurement** the discovery or estimation of the ratio of a magnitude of a quantity to a unit of the same quantity.

Note that Michell presents 'measurement' as the act rather than the result and 'measure' as a noun rather than a verb. Still, measurement qua result is a ratio of one magnitude to another, the latter one used as a standard.

Probably the first rigorous attempt to illuminate the nature of magnitude was Book V of Euclid's Elements. In any case his ideas have been very influential. One of his key ideas -- profound and powerful -- was that of ratio, which he defined as follows. "A ratio is a sort of relation in respect of size between two magnitudes of the same kind" (Euclid, Bk. V, Dfn. 3). [1]

Euclid's theory of ratios of magnitudes clearly influenced others over the centuries. Galileo's devotion to Euclid was intense. It strongly influenced Descartes as he created his revolutionary method of solving geometric problems by algebraic means. Newton explicitly defined number as the abstracted ratio of a quantity to a unit (Michell 1991, 32). Such influence continued with, among others, DeMorgan and later the eminent physicist James Clerk Maxwell, evidenced as follows.

> Quantities are abstract concepts possessing two main properties: they can be measured, that means that the ratio of two quantities of the same kind, a pure number, can be established by experiment; and they can enter into a mathematical scheme expressing their definitions or the laws of physics. A unit for a kind of quantity is a sample of that quantity chosen by convention to have the value 1. So that, as already stated by Clerk Maxwell,
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> Physical quantity = pure number x unit.
>
> This means that the ratio of the quantitative abstract concept to the unit is a pure number (ibid., 14)

> Every expression of a quantity consists of two factors or components. One of these is the name of a certain known quantity of the same kind as the quantity expressed, which is taken as the standard of reference. The other component is the number of times the standard is to be taken in order to make up the required quantity" (ibid., 33)

Algebraically then, Maxwell's formula is M = R x U, where M is magnitude, R is a real number and U is the unit. For many, but not all, kinds of units R is positive. R could be negative for rate of change in velocity, for example.

Divide both sides of the equation by U. The result is: M/U = R. This formula highlights the significance of *ratio*. It shows the intimate connection between ratios and real numbers. The left side is the ratio; it equates to a real number. Consider an example using inches:

15 in. = 15 x (1 in.)

If we divide both sides of this equation by 1 inch, then simplify the ratio 15/1 to 15, we get the identity 15 = 15, since inches in the numerator and denominator of the ratio cancel out. We would get a similar result no matter what unit of measurement is used.

Magnitudes so constructed are subject to mathematical operations, addition being the main one. For example, if we took a weight of 10 pounds and added 5 pounds to it, we would get 15 pounds. If we took a rope 30 feet long and cut off 5 feet, we would get a rope 25 feet long. If we took a gallon of milk and evenly split it four ways, we would get four quarts.

Magnitudes per one standard unit (of absolute scale) are easily transformed into magnitudes per a different standard unit simply by multiplying the former by a real number. For example, magnitudes in meters can be transformed to feet by multiplying by 3.2808399. The ratio of one magnitude to another is invariant of the unit used. For example, the ratio of the height of a rectangle to its width is the same, whether the magnitudes are in centimeters or inches.

I do not know about all of the many kinds of measurement there are, but generally measuring is done with some kind of *gauge*. Lengths are measured with a tape measure or ruler, weights with a scale or balance, time with a clock or watch, liquid volume with a marked container, electric current with an ammeter. The gauge is an *external* device. This makes determining magnitude largely a matter of observation. The user simply reads the result. As long as the user is competent with the gauge, and recognizing the natural limits of its precision, the reading is exempt from subjective influence. Any other competent user in identical or similar circumstances will come up with the same reading. These conditions are not met when it comes to subjective sensations.

Ordinal Ranking

According to Michell, prior to the mid-20th century measurement in psychology was understood in the classical manner. For example, J. M. Baldwin in his 1902 *Dictionary of Philosophy and Psychology* wrote about *measurement* as follows: "In order that a concept be measured as a mathematical quantity without ambiguity, it is essential that the special attribute by which we measure it conceivable as made up of discrete parts, admitting of unambiguous comparison *inter se*, with respect to their quality or difference in magnitude" (ibid., 185). Similarly H. C. Warren in his 1934 *Dictionary of Psychology* defined *measurement* as "The comparison of a quantitative datum of any sort with a fixed, enduring datum or standard of the same sort" (ibid., 185).

Then the situation in psychology changed with S. S. Stevens leading the way. Over a period of about 30 years, he said and wrote repeatedly, "Measurement is the assignment of numerals to objects or events according to a rule." Since the 1950s his definition of measurement has been the model for psychologists (ibid., 15).

Psychophysicists like Weber (1830 on) and Fechner (1860 on) made popular "quantitative psychology" with the development of algebraic formulas for linking measurements of external stimuli to psychological intensities or sensations. The latter -- relying on introspection – could be ranked with pretty small noticeable differences. Fechner took this as good enough to construe it as "measurement". Philosophically he conceived the mind and brain as one. There were strong objections -- the smallest noticeable difference is not constant on a linear or log scale and varies by person, the subject's sensation cannot be observed, and mind-body dualism. He responded vigorously, as did other psychophysicists later defending their own forms of quantifying. The debate went on for decades, with statistical theory entering the scene in the later years. Eventually the psychologists won the debate as far as they were concerned, in large part by ignoring the objections (ibid., 79-108).

Stevens, famous for his psychophysics of sound, continued the push for an alternative concept of measurement. He was influenced by and helped popularize operational definitions, by which any concept is nothing more than a set of operations and is synonymous with those operations. For example, if a subject, based only on his hearing, judges that one sound is twice another, that ratio 2 is as valid as any other ratio 2 (ibid., 162-90). Note, however, such ratio is not the result of dividing one number by another.

Stevens first called his psychophysical results "absolute judgment", then "numerical estimation", and finally "magnitude estimation." Most of this evolution was in the

1950's. His description of the method was that the subject brings the numbers with him, so to speak, and the experimenter needs only to provide the target stimuli to which the numbers are to be matched. Typical instructions to subjects were:

> You will be presented with a series of stimuli in irregular order. Your task is to tell how they seem by assigning numbers to them. Call the first stimulus any number that seems appropriate. Then assign successive numbers in such a way that they reflect your subjective impression (Stevens, 1971, 428).

Stevens even says, "Experience has shown that it is usually better *not to designate a standard"* [emphasis mine] (ibid.). Thus Stevens was well aware of the differences between psychophysical experiments and measurement in the harder sciences. But he had another aim -- to legitimize psychology as a science. Much of the prestige of the harder sciences comes from their being quantitative. Psychologists yearned for more prestige for their field and imitating the more quantitative sciences seemed to be a way. They wanted to regard psychological intensities or sensations as measurable. This was despite the existence of long-standing compelling arguments that they are not measurable in the classic manner. There are no exact numerical relationships between them and external stimuli, and there is no such thing as precise quantitative units of them. The external stimuli may be measurable, but that does not imply that psychological intensities or sensations are. Our quantifying them is of "less", "equal", or "more", but not of "how many times more or less" with precision and consistency. For example, physical pain is not measurable in the classical manner. Plausibly it is because it has so many dimensions – intensity, how localized it is and where, its duration, its quality (e.g. stabbing versus chronic), and the person's tolerance and attention.

I have no objection to psychophysics per se. Some things simply are not measurable. However, trying to redefine measurement is objectionable, and Stevens' definition opened the door far too wide. Suppose you agree to taste ten flavors of ice cream and then rank them with the numbers 1-10, 10 being for the flavor you like best and 1 being for the flavor you like least. This fits Stevens' definition. Does that mean you like flavor #10 twice as much as flavor #5 or ten times as much as flavor #1? Does that mean you like flavor #10 more than flavor #9 to the same degree you like flavor #7 more than flavor #6? One could alternatively rank them in reverse, with #1 as best and #10 as worse, but again the numbers assigned are not amenable to mathematical operations. If someone else agreed to taste the ten flavors and rank them, would such person rank each flavor with the same number you did? If you were to rank the same ten flavors in a few years, would you rank them in the same order? Except in rare circumstances, the answer to each of these questions is clearly 'no'. Such subjective influences do not occur with true measurement.

Note that the ranking could be done equally well with letters of the alphabet. All that is needed is a rule like assign A to the flavor you like best and J to the flavor you like least or vice-versa. Like letters, assigned numbers are no more than labels. Letters do just as well in expressing the fact that you like one flavor more or less than another. It is also clear that there is no standard quantitative unit of measurement here. There is nothing comparable to an inch, an hour, a liter, feet per second, and so on. If there were such a unit, a good name for it would be *yummy*.

To contrast the difference between true measurement and ranking using numbers, Michell uses the terms "discover" and "assign."

> Measurement is the attempt to discover real numerical relations (ratios) between things (magnitudes of attributes), and not the attempt to construct conventional numerical relations where they do not otherwise exist. The difference would be most dramatically seen if the attributes were not actually quantitative. Then there would be no ratios to discover, and measurement would be logically impossible. Nonetheless, numerical assignments according to some rule could always be made to the objects or events involved. This highlights the logical distinction: the making of numerical assignments may be many things (e.g., useful, convenient, rewarding), but true (or false) is not one of them. On the other hand, to claim that my room is four metres long is to assert something which is either true or false (ibid., 17).

This is an excellent distinction. The ice cream example is a case of trying to construct conventional numerical relations that do not otherwise exist.

True measurements are easily ranked, or ordered, because they have a basis in real numbers. Indeed, such measurements would be "well-ordered" in the mathematical sense. However, this does not work in reverse, since not all rankings are true measurements.

One can commonly find in psychology literature these days the following kinds of measurement or scaling. [2]

1. nominal – numbers on sports uniforms, social security numbers
2. ordinal – rank in graduation class or preference
3. interval – temperature, IQ
4. ratio – wealth, weight, distance

The first is mere labeling. For the second, differences are *relatively* meaningful – the differences are not based on a uniform unit - but the exact magnitudes are not. For the third, differences are *absolutely* meaningful - the differences are based on a uniform unit -- but the exact magnitudes are not. The fourth, of course, meets the criteria for classical measurement. *Teleological* measurement -- as Rand used the term – fits in the second category with "ordinal."

It should be clear by now that the first two fall far short of the criteria for the fourth. To pretend or suggest that they are equivalent to true measurement is unwarranted.

Rand Revisited

Ayn Rand showed a good understanding of measurement at times. For example: "Measurement is the identification of a relationship – a quantitative relationship established by means of a standard that serves as a unit" (Rand 1990, 7). However, she did not use the concept consistently, often lapsing into *teleological measurement* and *ordinal measurement*. These are constructs. In other words, she sometimes used m*easurement* in the manner of Stevens' definition, not the classical one.

Rand also used the term *unit* in two ways:

1. Member of a set or class: "A concept is a mental integration of two or more units which are isolated according to a specific characteristic(s) and united by a specific definition" (ibid., 10).

2. The basis of measurement: “Observe that measurement consists of relating an easily perceivable unit to larger or smaller quantities, then to infinitely larger or infinitely smaller quantities, which are not directly perceivable to man” (ibid., 8).

The second could be construed as a special case of the first, but they are not equivalent. In some places her usage could be taken either way: “The ability to regard entities as units is man’s distinctive method of cognition” (ibid., 6). “Units are things viewed by a consciousness in certain existing relationships” (ibid., 7). Whether or not these two meanings contributed to any confounding and ambiguity regarding “measurements omitted” is hard to say.

Regardless, a claim that it is *only* measurements that are omitted in concept formation implies that *every* kind of attribute is measurable. I think this has been shown to be false. Every kind of attribute would need a standard quantitative unit. If someone claims an attribute is measurable, the burden of proof is on him or her to say what this standard quantitative unit is.

The case is supported with the insightful, practical argument Norman Campbell gave in his1921 essay “Measurement”, paraphrased as follows. Some attributes and not all can be thus represented by numbers. When buying a sack of potatoes I may ask what it weighs and what it costs. To those questions I can expect a number in answer; it weighs 20 lbs. and costs 89 cents. But I may also ask of what variety the potatoes are, and whether they are good cookers; to those questions I shall not expect a number in answer. The dealer may possibly call the variety “No. 11” in somebody’s catalogue; but even if he does I shall feel that such use of number is not a real measurement, and is not of the same kind as the use in connection with weight or cost. What is the difference? Why are some attributes measurable and others not? It is this. Suppose I have two sacks of potatoes identical in weight, cost, variety, and cooking qualities; and I pour the two sacks into one so that there is only one sack of potatoes. This sack will differ from the two original sacks in weight and cost (the measurable attributes), but it will not differ from them in variety and cooking quality since these attributes that are not measurable (Campbell 1988, 1771-2).

It was noted above that Rand weakened the case for “omitting measurements” in connection with concepts of consciousness. Let us test a wider variety of concepts, even ones that are not concepts of consciousness, to see how well the claim of “measurements omitted” holds up more generally. In other words, we will try to follow Rand’s dictum of reducing a concept to its basis in particular facts (Rand 1990, 51).

Consider the concept *occupation*, in the sense of a job or career. Several particulars that might be treated as units in this concept are doctor, fireman, nurse, lawyer, teacher, computer programmer, civil engineer, truck driver, and salesman. They are similar in that each refers to particular kinds of activities the person does in order to earn an income. Such activities differ from case to case, but all such differences are *not* amenable to some kind of measurement. There is no standard quantitative unit that can be multiplied by a real number to derive a magnitude for each particular occupation and each kind of difference between these occupations. The differences in the activities performed are only *qualitative*.

Imagine a person assigning a number to each of the above particular occupations according to how he or she would rank each one as desirable, or their difficulty. It then might seem plausible that he or she has some “measurements” to omit. But what does such assignment gain as far as integrating these particulars to form the concept *occupation*? Nothing at all; indeed, the concept has already been formed. Again, letters of the alphabet would work just as well to rank them. The differences that need to be omitted are qualitative, not quantitative.

Consider ice cream again. Suppose a young child has eaten ice cream multiple times and has developed the concept ice cream. She is quite capable of identifying something as ice cream or not. If she has not yet experienced frozen yogurt or sorbet, she may mistake them for ice cream. Such a mistake is excusable and should not negate the judgment that she has a concept of ice cream. Has the child "omitted measurements"? I think not. Further suppose she is in still learning to count and has no idea what measurement is. She has attained her concept of ice cream by identifying its characteristic look, that it is cold, creamy and sweet, and its characteristic taste. The tastes differ among instances due to the particular flavor or how it was made, but there remains a characteristic ice cream taste due to the main ingredients. There are many differences among the various instances. Besides different flavors, some may include nuts, others not. Some are marbled, others not. But the child has treated each instance as a unit and integrated them into the concept ice cream. The most noticeable difference is flavor, which is a *qualitative* one. Again we would have to say most of these differences are *qualitative*. They are not of numerical attributes, hence not measurable, and it is logically impossible to have omitted measurements.

A defense of Rand's claim is that the child has *implicitly* omitted measurements. (On the other hand, some claim this part of her theory is very vague because she used "implicitly" so often with little explanation of the word). [3] Indeed, in my view this strategy does not bear scrutiny. It is fine to use "implicit" to mean a person does X, but the person only later becomes self-aware that he or she was doing X. It is fine to use "implicit measurement" to mean estimating length "by eyeball" as a substitute for using a tape measure. It is quite another matter to say that a person "implicitly" does X when the person lacks the ability to do X.

Studies of children provide plenty of evidence against the claim that a child who can recognize that a pencil is longer than a match and a stick is longer than a pencil is capable of measuring, even implicitly.

> The formation of concepts according to Rand's formula of measurement omission requires a dimensional understanding of isolated attributes. The dimensions must be accessible but might be scalable only ordinally (Rand, 1990, 31-33, 14-15). A child knows size as a dimension when he regards big and little as attributes of a single kind; part identities have then become organized as a dimensional kind. He knows size as an ordinal dimension when he regards bigger and littler as necessarily opposing direction of difference. Preschoolers have only a fragmented understanding of these quantitative relations (Boydstun 1990, 36). [4]

Fragmented indeed. Intermediate between being capable of knowing that one object is longer or shorter than another object and a grasp of measurement is *seriation*. This is Jean Piaget's term for the ordering or ranking of a few or several objects along a single dimension. His experiments with children, such as sorting several rods according to length, show that most children are not competent at the task until age 6-8 years. All the 4-year olds failed. Some had partial success, getting 2-4 rods in the correct order. Most 5-

and 6-year olds succeeded using trial and error. Only the 7- and 8-year olds were really competent – to take one rod at a time and put it into the correct place in the existing order without error (Gruber and Voneche, 1995, 385-6). Going beyond seriation to measurement requires using - not merely assigning - numbers to denote specific lengths based on an invariant standard quantitative unit.

Piaget and his colleagues tested children for their understanding of measurement in many ways. "For a long time children are content with visual comparison, and only later do they think of putting objects alongside each other to check their estimate, while the concept of a measuring rule which can provide a common measure arises still later" (Piaget, Inhelder and Szeminska 1960, 28). Children ages 4-5 years are usually in the first stage the authors describe, and the last stage is usually not reached until 7-8 years old. While the stages are described in much greater detail, with sub-stages, the following is a summary of the results.

1. Reliance on perception. No notion of common measure. Disregard of common measure even when instructed.
2. Common measure recognized. Partial success in measuring and partial reliance on perceptual judgment.
3. Full competency in measuring.

Some experiments tested a child's understanding of length or height or distance, often designed in such a way that measuring was the only reliable way of deciding that one line or object is longer or taller than or equal to another. Younger children were rarely capable of measuring even when given hints or training on how to do it (ibid., 27-66). Other experiments were a little more complicated. For example, the child was shown two lines with one or both being bent. Again the younger children showed no or little measurement ability, even when given hints or training about how to use a short strip of paper to "step off" a length. By age 7-8 most children were capable of measuring (Piaget 1960, 104-27). [5]

Such experiments show a huge difference between perception of quantity and the *concept* of measurement, even the practice of measurement. In conclusion, it seems to me that to claim a child when first capable of recognizing that a pencil is longer than a match, and a stick is longer than a pencil, is "implicitly measuring" is quite a leap. I would say the child has grasped "unit" in the first sense given above, but not the second.

Gellman and Williams offer reasons why children find learning fractions (ratios) -- required for measuring as said above -- difficult. Fractions are not part of the knowledge structure of the counting numbers. The children put new learning at risk because they rely so much on what they already know and how that knowledge is organized in their minds. All of us interpret new experiences using what we know, not what we *will* know. In Rand's words, "knowledge is contextual." Children are inclined to interpret input about fractions in light of what they think about counting – "numbers are what you get when you count." Gellman and Williams even claim that the idea of infinity is easier to learn than fractions. Infinity is consistent with the mathematical structure of counting – each number has a successor -- whereas fractions are not (Gellman and Williams, 1998, 617-18).

From an Objectivist perspective, when the child learns about measuring, she must integrate what she knew about counting with her new experience. She must do that to properly understand measuring and how it relates to counting. Rand's examples involved only linear measurements. To understand 2- or 3-dimensional measurement, one must also know some arithmetic. Two examples are speed or spatial volume.

Suppose a child knows that a dog or rabbit can run faster than her. Does that mean the child implicitly knows how to measure speed? Unlike length, speed explicitly involves two magnitudes, distance and time. Thus to implicitly measure speed, the child would need to implicitly measure both distance and time, then implicitly divide distance by time.

Next we examine Rand's own justification for "teleological measurement" being measurement.

> Measurement is the identification of a relationship – a quantitative relationship established by means of a standard that serves as a unit. Teleological measurement deals, not with cardinal, but with *ordinal* numbers – and the standard serves to establish a graded relationship of means to ends.
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> For instance, a moral code is a system of teleological measurement which grades the choices and actions open to man, according to the degree to which they achieve or frustrate the code's standard of value. The standard is the end, to which man's actions are the means (Rand 1990, 33).

In her examples that precede the above ("thought" and "emotion", quoted above), she appeals to measurement on the basis of psychological intensity. In other words she does what psychologists have done. She also uses "standard" here in two different ways. A standard for measurement is an invariant magnitude of the same kind that is being measured. Her use of "standard" here in regard to "teleological measurement" is a very different sort of standard. The implied sameness fails.

Ranking per se lacks the degree of precision that measurement has. In his definition of *number*, Jetton uses *definite* (meaning precise) to differentiate number from other kinds of quantification. Imprecise ways of quantifying – some, several, greater than, truckful, and so forth – are quantifying but without numbers (Jetton, 1990, 2). By analogy number is to measurement as ordinal ranking is to indefinite quantifying.

Rand again showed how far she stretched ordinary meanings when describing her theory of concepts as "mathematical." That is not wrong per se. Indeed, it is a normal part of learning and creativity. A great example of such stretching is her claim that love is measurable. She staked this on intensity -- using liking, affection, and romantic love. Accordingly it is ranking, not measuring, as explained above. Moreover, instances of love are differentiated by more than just intensity. They are also differentiated by other qualitative factors – like why a person, object, or activity is loved.

I do not know if she was aware of the debate going on in psychology or did this on her own. In any case the parallel exists. She also admitted to herself that her ideas about measurement and its omission were very tentative per the Journals of Ayn Rand. This was published after her death, and she probably did not intend it for publication (Rand, 1997, 710-12). In contrast, the tentative nature of her ideas is absent from ITOE.

Peikoff's OPAR

In OPAR (Peikoff, 1993) the author basically repeats the arguments that Ayn Rand made about measurement omission in ITOE. He reuses her examples of length and table.

He raves about her discovery of measurement omission, calling it “momentous” (ibid., 82) and a “seminal observation” (ibid., 83). He considers no wider variety of examples than did Rand. He comments:

> In her treatise, Miss Rand covers all the main kinds of concepts, relationships, and materials. In each case, she explains how the principle of measurement omission applies (ibid., 88).
>
> The process of measurement omission is performed by us by the nature of our mental faculty, whether anyone identifies it or not. To form a concept, one does not have to know that a form of measurement is involved; one does not have to measure existents or even know how to measure them. On the conscious level, one need merely observe similarities (ibid., 85).
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> The answer to the “problem of universals” lies in Ayn Rand’s discovery of the relationship between universals and mathematics. Specifically, the answer lies in the brilliant comparison she draws between concept formation and algebra (ibid., 89).

Peikoff’s idea of measurement does not conform to its scientific sense either. Given what Rand said was omitted for concepts of consciousness, the first quote from Peikoff is an exaggeration. The second quote is not very objective -- making an empirical claim but giving no empirical evidence. The paragraph is a non sequitur. If he could present some neurological or psychological evidence that this is a plausible hypothesis, then it would be time to listen. Meanwhile, it merely begs the question.

Consider the third quote from Peikoff. Why does he say that Rand’s solution rests on her comparison with *algebra*? Why is it not *measurement omission*, which both stress far more heavily? Nevertheless, here I think that Peikoff and Rand got it right. She compared concept formation to algebra as follows:

> The relationship of concepts to their constituent particulars is the same as the relationship of algebraic symbols to numbers. In the equation 2a = a + a, any number may be substituted for the symbol “a” without affecting the truth of the equation. For instance 2 x 5 = 5 + 5, or: 2 x 5,000,000 = 5,000,000 + 5,000,000. In the same manner, by the same psycho-epistemological method, a concept is used as an algebraic symbol that stands for any of the arithmetical sequence of units it assumes (Rand 1990, 18).

This is a great analogy, but she takes it too far in one respect. How are the various occupations considered above to be considered an “arithmetical sequence”? As she claims elsewhere, the symbol ‘a’ and a concept are both open-ended. However, that does not imply the units of a concept form an arithmetical sequence.

The most striking aspect of Rand’s analogy to me is the following parallel between (a) the letter ‘a’ and numbers and (b) a concept and its referents. Both relationships are between quite different kinds, at least when the references are external existents. In the first case, a letter represents something quite different in kind – numbers. In the second case, an idea represents something quite different in kind – external existents.

Another Perspective

There is another, more complex, way of looking at concepts algebraically. It seems to have potential for a richer explanation of concept formation and use. It needs more thought and may warrant another paper, but the following is a start. A general algebraic expression of an n-dimensional function is:

$$y = f\ (x1, x2, x3, \ldots, xn)$$

For example, a function for the area of a rectangle would be area = f (length, width). More specifically it would be area = length x width. Area and speed, mentioned earlier, are 2-dimensional on the right side, so n = 2. With n = 3 and a rectangular packing box, volume = f (length, width, height). While most people hesitate when n > 3, it is an easy step mathematically.

This same general expression can be extended to concepts, though the ability to use numbers often declines, for example:

bird = f (animal, two legs, wings, feathers/down, beak, fly)

This is more abstract since n = 6. More dimensions could be added to increase n. For a particular bird, all these variables have particular values, some of them quantitative and others only qualitative. In cognitive psychology the latter may be called *features* to note the difference (Kelley 1984, 7). Numerically the ‘y’ for bird may be restricted to 1 or 0, meaning ‘bird’ or ‘not a bird’ depending on the values on the right side. There could also be logically preceding functions for variables on the right for some concepts. But let us return to the main topic.

Alternative Proposals

If the reader is convinced by the above arguments, what could replace measurements omitted” in Rand’s description or definition of a concept? Some candidates are:

1. measurements or non-measurable qualities omitted
2. differences of content or measurement omitted
3. details omitted
4. differences of detail omitted
5. leaving out those particulars wherein they differ

I like the fourth best for several reasons:

1. It includes “differences” that contrasts it with “similar” in the first part of her description or definition of “concept.”
2. “Details” can cover both qualities and quantities.
3. It points to the fact that what we disregard in forming a concept are specific attributes (or further specifics about such attributes) of the units integrated by the concept that are not considered as fundamental. “Just give me the basics, not all the details” is a common request. “Details omitted” fits that request.
4. Rand amply described how regarding things as units and forming concepts gave us cognitive economy. “Details omitted” further appeals to the cognitive economy that concepts provide us.

I like the fifth a lot and took it from John Locke (Locke 1959, III, iii, 9).

This is not to deny that measurements are omitted with respect to some concepts, ones that have attributes that are quantitative to begin with, such as length and triangle. It simply recognizes that so many attributes are only (known to be) qualitative.

Rand's idea of similarity must also be challenged. She defined similarity in terms of sharing common dimensions, but differing in measure or degree (Rand 1990, 13). However, differences may be of type, not only measure or degree. In other words, they may be qualitative, not merely quantitative. For example, ice cream flavors differ in quality, not quantity. Airplane engines may be classified as propeller, jet, turbojet, or rocket. These types are similar in that they are means of propulsion for an airplane, but the difference is not a matter of measure or degree.

I even propose that quality is logically prior to quantity. A measurement is *of something*, that something being some quality.

Notes

1. Euclid conceived ratios as one whole number to another. The decimal system of notation we use today did not come into existence until more than 1700 years later. Book V does not even use numbers for magnitudes. It uses one line segment relative to another, using letters like A and AB to designate line segments.

2. Probability and statistics are often used in psychology, too. It is not exactly classical measurement, but it is mighty close when the numbers come from counting or measuring – not mere assignment – and computing ratios.

3. Brian Register held that Ayn Rand was quite vague about her use of "implicit" in an earlier edition of this journal (Register 2000, 224-25). Robert Campbell endorsed Register's view and made further comments on the vagueness of her usage (Campbell 2000, 214-15).

4. Boydstun buys into Rand's attempt to consider any ordinal ranking as a form of measurement (Boydstun 2004, 8). Of course, I do not.

> Like the common dimension for strength, it is a dimension consisting of nothing more than various substitution dimensions. The measurable dimension of *property* of a solid will be the hardness dimension or the tensile-strength dimension or the principal-curvature dimensions or . . . There is no single, common measure of *property* of a solid that all specific properties of solids have in common. Rand supposed in error that there were, for she supposed it always the case that there is some same, common measurable dimension supporting the conceptual common denominator for any superordinate concept. That supposition is here rejected, and measurement-omission analysis of superordinate concepts is here corrected in this respect (ibid., 13).

Here he seems to agree with me -- that Rand erred in supposing there is always some same measurable dimension supporting the conceptual common denominator for any super-ordinate concept. I take it further -- that it is not only super-ordinate concepts.

> But there is one form of challenge that is invalid, and I want to draw attention to this fallacy, which has required a long struggle for me to overcome. That is a fallacy I insinuated in Boydstun 1990, 33–34. It says that because preschoolers do not possess—not even tacitly—mathematical understanding sufficient to be forming their concepts using a principle of measurement-omission, their concepts do not bear analysis in terms of measurement-omission. That is the fallacy of confusing genesis with analysis (ibid., 24).

What exactly he means by this fallacy is not clear to me, but in my view it is descriptive of Rand's oversight. Being able to analyze a concept on a certain criteria later on does not imply that criteria were material when the concept was formed. Also, here Boydstun seems to contradict the above excerpt from page 13, at least when the child forms a super-ordinate concept.

5. Piaget also tested children's ability to judge whether quantities of liquids differed or were the same. The liquids were in different containers so shaped that one could not estimate their ratios by direct perception. There were extra empty containers that could be used to determine if the liquid in one container was less than, equal to, or greater than the liquid in another container. In the first case, no hint is given to the child about using one of the extra containers as a unit to measure. In the second case, the child is instructed in such use. Similar to length the children were not capable of measuring until about age 8. These experiments were about volume rather than length, but the results again show the large difference between perception of quantity and the *concept* of measurement (Gruber and Voneche 1995, 330-4).

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